Supplementary Materials for

Soft magnetic skin for super-resolution tactile sensing with force self-decoupling

Youcan Yan, Zhe Hu, Zhengbao Yang, Wenzhen Yuan, Chaoyang Song, Jia Pan*, Yajing Shen*

*Corresponding author. Email: jpan@cs.hku.hk (J.P.); yajishen@cityu.edu.hk (Y.S.)

Published 24 February 2021, Sci. Robot. 6, eabc8801 (2021)
DOI: 10.1126/scirobotics.abc8801

The PDF file includes:

Text S1. Formula derivation.
Text S2. Sensor responses when touching ferromagnetic materials.
Fig. S1. Magnetization loop of the flexible magnetic film.
Fig. S2. The sensing range and sensitivity of the sensor under different magnetic period of the flexible magnet.
Fig. S3. Sensitivities of the sensor for the normal and shear forces.
Fig. S4. Response time of the sensor (15 ms).
Fig. S5. The simulation result of the magnetic flux density distribution of the flexible magnet.
Fig. S6. Relationship between the magnetization manner of the flexible magnet and the corresponding force decoupling ability of the sensor.
Fig. S7. Receptive field (x directional response $B_x$) of all nine taxels.
Fig. S8. Receptive field (y directional response $B_y$) of all nine taxels.
Fig. S9. Receptive field (z directional response $B_z$) of all nine taxels.
Fig. S10. Localization procedure using responses of two neighboring taxels.
Fig. S11. Localization error using responses of every individual taxel as the inputs of the neural networks, where the error bars represent SDs of the localization error across 10 trials.
Fig. S12. Hysteresis curve of the sensor output during a cycle of loading and unloading (0 to 230 kPa).
Fig. S13. Repeatability test of the sensor over 30,000 cycles.
Fig. S14. Signal-to-noise ratio model of the sensor.
Fig. S15. Schematic illustration of the data collection process.
Fig. S16. Measurements of the magnetic strength on the strong and weak sides of the magnetic film.
Other Supplementary Material for this manuscript includes the following:

(available at robotics.sciencemag.org/cgi/content/full/6/51/eabc8801/DC1)

Movie S1 (.mp4 format). Real-time contact position tracking of a rolling ball.
Movie S2 (.mp4 format). Adaptive grasping of an egg with dynamic disturbance.
Movie S3 (.mp4 format). Adaptive grasping of a water-filling bottle.
Movie S4 (.mp4 format). Teleoperated needle threading.
Movie S5 (.mp4 format). Sensor responses when touching ferromagnetic materials.
Text S1. Formula derivation.

A) Magnetic flux densities below the flexible magnet

Imagine a planar structure of thickness \( d \) lying in the \( x\text{-}y \) plane, and the upper and lower surfaces are at \( z = 0 \) and \( z = -d \), respectively. Suppose the magnetization is the superposition of two sinusoids in quadrature:

\[
M_x = M_0 \sin(kx), M_z = M_0 \cos(kx), M_y = 0 \quad \text{(S1)}
\]

where \( k \) is the wavenumber and \( M_0 \) is the maximum magnitude of each component. According to (37), the magnetostatic scalar potential \( \varphi \) that is above, inside and below the sheet can be derived respectively as:

\[
\varphi_{\text{above}} = 0 \quad \text{(S2a)}
\]

\[
\varphi_{\text{inside}} = \frac{M_0}{k} \left( e^{kz} - 1 \right) \cos(kx) \quad \text{(S2b)}
\]

\[
\varphi_{\text{below}} = \frac{M_0}{k} \left( 1 - e^{kd} \right) e^{kz} \cos(kx) \quad \text{(S2c)}
\]

Remarkably, the scalar potential above the sheet (weakside) \( \varphi_{\text{above}} \) is zero, which provides the theoretical possibility of producing one-sided magnetic flux. In MKS-SI units, the flux density \( B \) is proportional to the sum of the magnetic field \( H \) and the magnetization \( M \), thus:

\[
B = \mu_0 (H + M) \quad \text{(S3)}
\]

where \( \mu_0 \) is the permeability of free space, which equals \( 4\pi \times 10^{-7} \text{ H\cdot m}^{-1} \). The magnetic field \( H \) is the negative gradient of the scalar potential \( \varphi \), and the magnetization \( M \) outside the magnetic material is zero, thus:

\[
H = -\nabla \varphi \quad \text{(S4)}
\]

\[
M = 0 \quad \text{(S5)}
\]

Combining Eq. (S3)-(S5), we can derive the magnetic flux density \( B_x \) (along \( x \)-axis) and \( B_z \) (along \( z \)-axis) below the sheet as:

\[
B_x = \mu_0 \frac{\partial \varphi_{\text{below}}}{\partial x} = -M_0 \left( 1 - e^{kd} \right) e^{kz} \sin(kx) \quad \text{(S6)}
\]

\[
B_z = \mu_0 \frac{\partial \varphi_{\text{below}}}{\partial z} = M_0 \left( 1 - e^{kd} \right) e^{kz} \cos(kx) \quad \text{(S7)}
\]

According to Eq. (S6) and (S7), we can calculate the overall (resultant) magnetic flux density \( B(x,z) \) and the ratio \( R_B(x,z) \) of \( B_x \) and \( B_z \) at any point \( (x,z) \) in the \( x\text{-}z \) plane of a planar Halbach magnet as follows:

\[
B(x,z) = \sqrt{B_x^2 + B_z^2} = M_0 \left( 1 - e^{kd} \right) e^{kz} \quad \text{(S8)}
\]

\[
R_B(x,z) = \tan \alpha(x,z) = \frac{B_x}{B_z} = \tan(kx) \quad \text{(S9)}
\]
Then we can calculate the magnetic strength change ($\Delta B\%$) under the displacement load along z-axis ($\Delta Z$) as follows:

$$
\Delta B(\%) = \left[ \frac{B - B_0}{B_0} \right] \times 100\% = \left[ \frac{M_o \left( 1 - e^{ikd} \right) \left( e^{-k(h-\Delta Z)} - e^{-kh} \right)}{M_o \left( 1 - e^{ikd} \right) e^{-kh}} \right] = e^{k\Delta Z} - 1 \tag{S10}
$$

where $B$ and $B_0$ are magnetic strength sensed by the Hall sensor with and without the external force, and $h$ is the thickness of the elastomer layer. It is worth noting that $\Delta B\%$ is independent of $h$, which implies that the sensitivity of the sensor is proportional to $h$. Here, the sensitivity is defined as $S = \Delta(\Delta B\%)/\Delta P$, where the pressure change $\Delta P = E(\Delta Z/h)$ and $E$ is the elastic module of the silicone.

**B) Fitting curves of the normal force $F_z$ and the shear force $F_x$**

As shown in Fig. 4A, if the flexible magnet has a “rigid” (overall) deformation from ($x_0, z_0$) to ($x_1, z_1$) under the external force $F$, we can calculate the magnet’s displacement $\Delta x$ along $x$ direction and $\Delta z$ along $z$ direction from the change of $R_\theta(x)$ and $B(z)$ respectively based on Eq. (S8) and (S9):

$$
\Delta x = x_1 - x_0 = \frac{1}{k} \left[ \arctan \left( \frac{B_x(x_1, z_1)}{B_z(x_1, z_1)} \right) - \arctan \left( \frac{B_x(x_0, z_0)}{B_z(x_0, z_0)} \right) \right] \tag{S11}
$$

$$
\Delta z = z_1 - z_0 = \frac{1}{k} \ln \left( \frac{\sqrt{B_x(x_1, z_1)^2 + B_z(x_1, z_1)^2}}{\sqrt{B_x(x_0, z_0)^2 + B_z(x_0, z_0)^2}} \right) = \frac{1}{k} \ln \left( \frac{B(x_1, z_1)}{B(x_0, z_0)} \right) \tag{S12}
$$

Consequently, the shear component $F_z$ and normal component $F_x$ of the external force $F$ can be calculated with the magnet’s displacement $\Delta x$ and $\Delta z$, respectively, since the applied force and the corresponding displacement of the elastic material are linearly correlated according to Hooke’s Law:

$$
F_x = S \cdot (G \cdot \gamma) = S \cdot G \cdot \frac{\Delta x}{h} \tag{S13}
$$

$$
F_z = S \cdot (E \cdot \varepsilon) = S \cdot E \cdot \frac{\Delta z}{h} \tag{S14}
$$

where $S$ is the contact area, $\gamma$, $\varepsilon$, $G$, $E$ and $h$ is the shear strain, normal strain, shear modulus, elastic modulus and thickness of the elastomer layer, respectively. For isotropic elastic materials, $G = E/(2(1+V))$, where $V$ is the Poisson’s ratio of the material. As the center of the magnetic sensor aligns with the center of the north magnetic pole in the sensor, $B_z(x_0, z_0)$ in Eq. (S11) therefore equals zero. Substituting $\Delta x$ and $\Delta z$ in Eq. (S13) and (S14) and introducing four compensation coefficients, we can get:

$$
F_x = \frac{c_1 \cdot S \cdot G}{k \cdot h} \arctan \left( \frac{B_x(x_1, z_1)}{B_z(x_1, z_1)} \right) + b_1 \tag{S15}
$$

$$
F_z = \frac{c_2 \cdot S \cdot E}{k \cdot h} \ln \left( \frac{B(x_1, z_1)}{B(x_0, z_0)} \right) + b_2 \tag{S16}
$$
where $c_1$ and $c_2$ are compensation coefficients for elastic modulus and shear modulus, respectively since the overall elastic/shear modulus of the sensor is a resultant value of the modulus of the top or middle layers (as well as some invisible defects during the fabrication process). The $b_1$ and $b_2$ are systematic bias to ease the calibration of the sensor in different applications.

**Text S2. Sensor responses when touching ferromagnetic materials.**

In our sensor design, the magnetic film is magnetized sinusoidally (in Halbach arrays) with multiple alternate north-south poles, so that the magnetic field is strengthened on the one side (strong side, towards the Hall sensor) and canceled to nearly zero on the other side (weak side, towards the object to be touched) (see Eq. (S2)). The magnetic strength on the two sides of the magnetic film is as shown in fig. S1. We observe that the maximum magnetic strength on the weak side is only 2.2mT, which is 16.6% of that on the strong side (13.4mT) and 4.4% of that on the surface of a small permanent magnet (~50mT for the N35 NdFeB magnet that is of 2mm in thickness and 25mm in diameter). That is to say, there would be a very weak attraction between the surface (weak side) of the sensor and “soft” ferromagnetic materials (like iron). To verify this conclusion, we conducted an experiment that touching an iron-made paper clip with the two sides of the magnetic film respectively (see movie. S5). It is found that the strong side of the film can easily attract the paper clip while the weak side cannot, indicating that the magnetic strength on the weak side is indeed smaller than that on the strong side and thereby there is much weaker attraction between the weak side and the ferromagnetic items.

In addition, when an unmagnetized metal material touches the proposed sensor, there is no obvious distortion of the magnetic field lines. This is because the magnetic field above the sensor surface is too weak to magnetize the metal, and thereby there is no magnetic interaction between the unmagnetized metal and the tactile sensor. However, when the metal is magnetized by a permanent magnet or electromagnet, it would distort the magnetic field of the sensor slightly if the metal is a “soft” ferromagnetic material (like iron) with low remanence, or significantly if the metal is a “hard” ferromagnetic material (like alnico and ferrite) with high remanence. To verify this, an iron-made paper clip was put on the sensor surface when it is not magnetized and magnetized, respectively (movie. S5). We observe that the presence of the unmagnetized paper clip did not affect the sensor outputs, while the presence of the magnetized one slightly distorted the sensor outputs, which is consistent with the above analysis.

Moreover, permanent magnets and electromagnets would disrupt the sensor outputs. However, such disruption is temporary and would not damage the magnetization pattern of the film as long as the external magnetic field is no larger than the intrinsic coercivity of the NdFeB powders of the magnetic film (see fig. S1), which is 7779.15Oe or around 778mT. As shown in movie. S5, the sensor outputs are significantly disrupted when a N35 NdFeB permanent magnet (2mm in thickness and 25mm in diameter) approaches and then get back to the normal as the magnet is removed. This is because the magnetic strength on the surface of the N35 NdFeB magnet is only ~50mT (measured by a Gaussmeter), which is far smaller than 778mT and thus cannot demagnetize the magnetization pattern of the film. Therefore, there is little chance that touching objects in our daily life could damage the magnetization pattern of
the sensor, since the magnetic field around us is normally far smaller than 778mT.

**Characterization of the flexible magnetic film**
The magnetization loop (or \( M-H \) loop) of the flexible magnetic film was measured with MPMS3 (Quantum Design Inc., San Diego, USA) as shown in Fig. S1, from which we can read that the remanent magnetization \( M_r \) is 120.335emu/cm\(^3\) at \( H = 0 \) and the intrinsic coercivity \( H_{ci} \) is \(-7779.15\)Oe at \( M = 0 \). Then, the remanent induction \( B_r \) can be calculated by multiplying \( 4\pi \) to \( M_r \) in CGS units, which gives us 1512.17Gs (1emu/cm\(^3\) = 1Gs), and as the flexible magnet is “soft” magnet, we consider the coercivity \( H_c \) of the magnet to be approximately equal to the intrinsic coercivity \( H_{ci} \).

![Fig. S1. Magnetization loop of the flexible magnetic film.](image)

**Relationship between the magnetic period of the magnetic film and the sensor range and sensitivity**
The maximum sensing range of normal force depends on the maximum allowable deformation of the sensor in the normal direction, which is equal to the maximum elastomer thickness \( h \). Here the maximum thickness \( h \) is defined by the distance away from the bottom surface of the magnetic film to the place where the attenuation of the magnetic strength is 98% of the maximum magnetic strength. The maximum sensing range of the shear force is defined as the half period (\( T/2 \)) of the magnetic field according to \( R_B = \tan(2\pi/T\cdot x) \), where \( T \) is the period of \( R_B \). The sensitivity of normal force is defined as \( S_1 = \Delta(\Delta B\%)/\Delta P = e^{(k\cdot \Delta z\cdot 1)}/(E_z\cdot \Delta z) \) where \( k \) is the wavenumber (equals \( 2\pi/T \)), \( \Delta z \) is the deformation in the normal direction and \( E_z \) is the elastic modulus of the elastomer layer in the normal direction. The sensitivity of the shear force is defined as \( S_2 = \Delta(\Delta R_\beta\%)/\Delta P = \tan(k\cdot \Delta x)/(E_x\cdot \Delta x) \) where \( \Delta x \) is the deformation in the shear direction and \( E_x \) is the elastic modulus of the elastomer layer in the shear direction (suppose \( E_x\cdot E_z= E_c\)). Here the \( \Delta z \) and \( \Delta x \) are 0.1mm; \( S_1 \) and \( S_2 \) are the sensitivities of the sensor at the initial deformation stage. \( T_1 \) and \( T_2 \) are magnetic periods that can balance the sensing range and sensitivity of the sensor for the normal force and the shear force, respectively. And to balance the sensor performance in normal and shear directions, we chose the magnetic period as \( T = 6 \)mm that is an integer value (for ease of magnetization with a ready-made machine) between \( T_1 \) (5.81mm) and \( T_2 \) (6.24mm) as shown in Fig. S2.
Fig. S2. The sensing range and sensitivity of the sensor under different magnetic period of the flexible magnet. (the magnetic period $T$ of the magnetic film is defined as the center-to-center distance between two neighboring magnetic poles of the same polarity).

Experimental measurements of the sensor sensitivities in the normal and shear directions
The sensitivity is 0.01kPa$^{-1}$ ($P \leq 120$kPa) for the normal force and 0.1kPa$^{-1}$ ($P \leq 10$kPa) to 0.27kPa$^{-1}$ (10kPa<$P \leq 16$kPa) for the shear force. However, the sensitivities in both the normal and shear directions can be adjusted to accommodate different applications by designing the sensor parameters.

Fig. S3. Sensitivities of the sensor for the normal and shear forces.

Response time of the sensor
The instant response time of the sensor is measured by calculating the dropping time of the sensor signals when the external load is quickly removed from the sensor. Here the sensor signal’s sampling rate is 130Hz.
Fig. S4. Response time of the sensor (15 ms).

Finite element analysis of the magnetic flux density distribution of the flexible magnet
The simulation result (in ANSYS Maxwell 19.1) of the magnetic flux density distribution of the flexible magnet is shown in Fig. S5, where the remanent induction $B_r$ was set to 1512.17Gs and the coercivity $H_c$ was set to $-7779.15\text{Oe}$ in the simulation environment as derived in Fig. S1.

Fig. S5. The simulation result of the magnetic flux density distribution of the flexible magnet.

Comparison of two different magnetization methods and the corresponding force decoupling ability of the sensor
The force decoupling ability of the sensor can be designed by adjusting the magnetization pattern of the flexible magnet. In this manuscript, the flexible magnet is sinusoidally magnetized so that the overall magnetic directions (or polarizations) are in a stripe-like manner as shown in the left side of fig. S6, which results a 2-axis ($x$-$z$) force decoupling ability since the magnetization strength in $y$-axis is zero. However, the force decoupling ability of the sensor can be theoretically extended to 3-axis ($x$-$y$-$z$) by sinusoidally magnetizing the flexible magnet along the radial direction, so that the polarization pattern of the flexible magnet is in a concentric ring-like manner as shown on the right side of fig. S6. It is worth noting that in order to generate the desired magnetization pattern, a customized electrical magnetizer with proper coil design is required.
**Fig. S6. Relationship between the magnetization manner of the flexible magnet and the corresponding force decoupling ability of the sensor.**

**Receptive field of all nine taxels (S1-S9)**
For the characterization of the super-resolution property, the flexible magnet (18mm x 18mm) was sampled at 0.2mm spacing in both the x direction and the y direction, resulting 8100 sampling points (90 x 90) in total. When the flexible magnet was pressed at each sampling point, the sensor responses of all nine taxels in three directions (x, y, and z) were measured as shown in Fig. S7, Fig. S8 and Fig. S9, respectively.
Fig. S7. Receptive field (x directional response $B_x$) of all nine taxels.
Fig. S8. Receptive field (y directional response $B_y$) of all nine taxels.
Fig. S9. Receptive field ($z$ directional response $B_z$) of all nine taxels.

Localization procedure of tactile super-resolution with considering responses of two neighboring taxels
To precisely locate the contact position on taxel S5, two neural networks with the same architecture are employed for estimating the accurate X location and Y location, respectively. The responses of both the taxel S5 and its neighboring taxels (S6 in $x$ direction and S2 in $y$ direction) are considered as inputs of the neural networks.
Fig. S10. Localization procedure using responses of two neighboring taxels.

Localization error without considering the responses of neighboring taxels
For tactile super-resolution, when the inputs of the neural networks are replaced by the magnetic flux densities of every individual taxel, the localization error increases compared to that when considering the responses of neighboring taxels (Fig. 4A). To be specific, the best localization error is 0.09mm for $X$ location and 0.11mm for $Y$ location on average (six inputs) when considering the responses of neighboring taxels, which are much smaller than that (0.12mm for $X$ location and 0.41mm for $Y$ location) with three inputs when just considering the response of every single taxel.

Fig. S11. Localization error using responses of every individual taxel as the inputs of the neural networks, where the error bars represent SDs of the localization error across 10 trials.

Hysteresis curve of the sensor output during a cycle of loading and unloading (0~230kPa).
The hysteresis curve of the sensor is as shown below, which plots the sensor output during a cycle of loading and unloading (0~230kPa). We observe that the overall hysteresis of the sensor is relatively small, and the maximum hysteresis error is 3.5% (increased 143.15uT at 117.6kPa) during the unloading stage.
Fig. S12. Hysteresis curve of the sensor output during a cycle of loading and unloading (0 to 230 kPa).

Repeatability test illustrating the stability of the sensor response over 30,000 cycles
The sensor response can be reproduced for tens of thousands number of cycles as shown in fig. S13. After applying a pressure of 110 kPa and releasing to zero for more than 30,000 cycles (duration of one cycle: 6s), the minimum and maximum values of measured magnetic flux density $B_z$ increased by 4.1% and 0.08%, respectively, suggesting that the proposed sensor is robust and repeatable for long-term usage.

Fig. S13. Repeatability test of the sensor over 30,000 cycles.

Signal-to-noise ratio (SNR) model of the sensor
The signal-to-noise ratio (SNR) of the sensor is high, as shown in fig. S14, which is defined as:

$$SNR = \frac{\mu - \mu_0}{\sigma_0}$$  \hspace{1cm} (S17)

$$SNR_{dB} = 20\log_{10}\left(\frac{\mu - \mu_0}{\sigma_0}\right) dB.$$  \hspace{1cm} (S18)
where $\mu$ and $\mu_0$ are the mean values of the sensor output when loaded and not loaded, respectively, and $\sigma_0$ is the standard deviation when not loaded. It is found that the sensor has a signal-to-noise ratio (SNR) of 51 (or 34dB) at the pressure of 11.8kPa, and greater SNRs can be obtained in scenarios with higher pressure.

**Fig. S14. Signal-to-noise ratio model of the sensor.**

**Schematic illustration of the data collection process**

As shown in fig. S15, the tactile data are collected from the Hall sensor with an Arduino Mega 2560 via I²C protocol, and then sent to a laptop via USB cable for further processing. We note that only four wires are required for the I²C communication between an MCU (Microprogrammed Control Unit) and multiple Hall sensors (embedded on a PCB): SDA (Serial Data), SCL (Serial Clock), VCC (Power) and GND (Ground), because each I²C slave device (Hall sensor here) has a unique physical address and can be visited independently via a single I²C bus. The schematic circuit diagram of the PCB is as shown on the left side of the fig. S15, where the MLX90393 chip (16 pin QFN package) is powered by the VCC/GND ports and queried by the SDA/SCL ports of the PCB. R1 and R2 are pull-up resistances (10kΩ) required by the I²C protocol, and C0 and C1 are decoupling capacitors for canceling the AC noise from DC signals.

**Fig. S15. Schematic illustration of the data collection process.**

**Measurements of the magnetic strength on the two sides of the magnetic film**
The magnetic film of our sensor is magnetized sinusoidally (in Halbach arrays) with multiple
alternate north-south poles. Thus, the magnetic field is strengthened on the one side (strong side) and canceled to zero (ideally) on the other side (weak side) as quantitively defined in Eq. (S2). However, it is hard to perfectly magnetize the magnetic film with a sinusoidal magnetization pattern in practice, so the magnetic strength on the two sides of the magnetic film (as shown in fig. S16) is slightly different from the theoretical results (Eq. (S2)). We observe that the maximum magnetic strength on the weak side is 2.2mT (not zero as in the ideal case), which is 16.6% of that on the strong side (13.4mT), and the magnetic strength on both sides of the magnetic film decays rapidly when leaving the surface of the film.

![Graph showing magnetic strength on the strong and weak sides of the magnetic film.](image)

**Fig. S16.** Measurements of the magnetic strength on the strong and weak sides of the magnetic film.